Pre-class Warm-up!!!

In the class on Monday, did we learn what the terms consistent and inconsistent mean, when applied to a linear system of equations?

a. Yes

b. No

Can anyone even remember what we did on Monday anyway?

Section 3.2: Matrices and Gaussian elimination

We learn:
How to solve systems of linear equations in exactly the same way as before, but changing the notation.
A theorem that row operations do not change the solution set.

Vocabulary:

- Matrix, entries, size of the matrix
- (Augmented) coefficient matrix
- Elementary row operations ✓
- Row equivalent
- Echelon matrix
- Back substitution ✓, free variables, leading variables
- Gaussian elimination algorithm

What are matrices, the size of a matrix, the labeling of the entries?

A matrix is a rectangular array of numbers, like

$$\begin{bmatrix} -2 & 0 & 3 \\ 4 & -17.1 & 32 \end{bmatrix}$$

I like square brackets

This is a 2×3 matrix (2 rows 3 columns)

This is the $(2,3)$ entry of the matrix

Sometimes we might write the matrix as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = (a_{ij})$$

where $a_{11} = -2$
$a_{21} = 4$

Elementary operations from Section 3.1

1. Multiply an equation by a non-zero scalar.
2. Switch two equations.
3. Add a multiple of one equation to another.

Solve, using elementary operations

$$\begin{aligned} 2y + 3z &= 7 \\ 2x + 4y + z &= -1 \\ x + 3y + 2z &= 3 \end{aligned}$$

eqn 1 $\longleftrightarrow$ eqn 3

$$\begin{aligned} x + 3y + 2z &= 3 \\ 2x + 4y + z &= -1 \\ 2y + 3z &= 7 \end{aligned}$$

eqn 2 $\longrightarrow$ eqn 2 − 2 eqn 1

$$\begin{aligned} x + 3y + 2z &= 3 \\ 0 - 2y - 3z &= -7 \\ 2y + 3z &= 7 \end{aligned}$$

eqn 3 → eqn 3 + eqn 2

$$\begin{aligned} x + 3y + 2z &= 3 \\ -2y - 3z &= -7 \\ 0 &= 0 \end{aligned}$$

Back substitution: z can be anything

$y = \frac{1}{2}(7 - 3z)$, $x = 3 - 3y - 2z$

$x = 3 - \frac{3}{2}(7 - 3z) - 2z = \frac{-15}{2} + \frac{5z}{2}$.

General solution: $(x, y, z) = \left(\frac{-15 + 5z}{2}, \frac{7 - 3z}{2}, z\right)$

Or we can use matrix notation:

$$\begin{bmatrix} 0 & 2 & 3 & 7 \\ 2 & 4 & 1 & -1 \\ 1 & 3 & 2 & 3 \end{bmatrix} \xrightarrow{\text{row 1} \leftrightarrow \text{row 3}} \begin{bmatrix} 1 & 3 & 2 & 3 \\ 2 & 4 & 1 & -1 \\ 0 & 2 & 3 & 7 \end{bmatrix}$$

$$\xrightarrow{(2) \to (2) - 2(1)} \begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & -2 & -3 & -7 \\ 0 & 2 & 3 & 7 \end{bmatrix} \xrightarrow{(3) \to (3) + (2)} \begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & -2 & -3 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Elementary operations for matrices.

1. multiply a row by a non-zero Scalar
2. Switch two rows
3. Add a multiple of one row to another.

Like questions 11 - 22:
Use elementary row operations to transform each augmented coefficient matrix to echelon form. Then solve the system by back substitution.

$2x + 3y + z = 1$
$4x + 6y + 2z = 2$
$6x + 9y + 4z = 3$

Solution

$$\begin{bmatrix} 2 & 3 & 1 & 1 \\ 4 & 6 & 2 & 2 \\ 6 & 9 & 4 & 3 \end{bmatrix} \xrightarrow[\textcircled{3} \to \textcircled{3} - 3\textcircled{1}]{\textcircled{2} \to \textcircled{2} - 2\textcircled{1}}$$

$$\begin{bmatrix} 2 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{\textcircled{2} \leftrightarrow \textcircled{3}} \begin{bmatrix} 2 & 3 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$z = 0$, y is "free variable"

$2x + 3y + z = 1$, $x = \dfrac{1 - 3y - z}{2} = \dfrac{1-3y}{2}$

General solution: $(x, y, z) = \left(\frac{1-3y}{2}, y, 0\right)$

Leading entries, x and z are leading variables.

y is a free variable

Echelon form: 1. All zero rows are at the bottom

2. In row the leading entry occurs in a later column than the leading entries in earlier rows.

New vocabulary: leading entries, free variables.

Like questions 11 - 22:
Use elementary row operations to transform each augmented coefficient matrix to echelon form. Then solve the system by back substitution.

$$
\begin{aligned}
2x + 3y &= 1 \\
4x + 6y &= 2 \\
6x + 9y &= 4
\end{aligned}
$$

Question:
How many solutions does the system on the left have?

a. One

b. None

c. Infinitely many

Echelon form

1. All zero rows are at the bottom.
2. In each non-zero row the leading entry is in a later column the leading entries of earlier rows.

Which matrices are in echelon form?

a. $\begin{bmatrix} 0 & 0 & 3 & 4 & 5 \\ 0 & 1 & 7 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ Yes No ✓ consistent.

b. $\begin{bmatrix} 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ Yes No ✓ consistent

w x y z

$z = 0$

$y = \frac{5}{3}$ w, x : free

c. $\begin{bmatrix} 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ ✓Yes No

inconsistent

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For what values of k does the system have a unique solution?

$3x + 2y = 0$
$6x + ky = 0$

a. $k = 4$
b. k not equal to 4 ✓
c. There are no such values of k
d. All values of k
e. $k = 2$

Solution

$② \to ② - 2①: \quad 3x + 2y = 0$
$\qquad\qquad\qquad\quad (k-4)y = 0$

If $k=4$ then y is a free variable, and can be chosen to be anything. We get inf. many solutions.

When $k \neq 4$ then $y=0$ is forced so $x=0$. Unique solution $(0,0)=(x,y)$.

On the Canvas home page it now says Review for Exams. You can find a past exam and extra HW questions.

Theorem 1 on page 160
If the augmented coefficient matrices of two linear systems are row equivalent, then the two systems have the same solution set.

row equivalent : we can get from one to the other by doing elementary row operations.